Conductivity rules in the Fermi and charge-spin separated liquid

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Ioffe-Larkin rule applies for the pure charge-spin separation regardless of its dimensionality. Here, an extension to this rule as a result of the coexistence of spinon, holon and electron as a single entity in the 2-dimensional (2D) system is derived, which is also in accordance with the original rule.

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1. Introduction

The coexistence of Fermi (electron) and charge (holon)-spin (spinon) separated (F-CSS) liquid in 2D high-T_c cuprate superconductors have been reported in order to explain the T dependence of magneto-thermo-electronic transport properties [1, 2]. Such coexistence strictly requires the ab-plane and c-axis resistivity models (σ^{-1}) in the form of

$$\begin{aligned} \sigma_{ab}^{-1} &= \sigma_s^{-1} + \sigma_h^{-1} + \sigma_{s+h\rightarrow e}^{-1} = \sigma_s^{-1} + \sigma_h^{-1} + \gamma\sigma_e^{-1} \\ \sigma_c^{-1} &= \sigma_e^{-1} + \sigma_{e\rightleftharpoons s+h}^{-1} = \sigma_e^{-1} + \beta[\sigma_h^{-1} + \sigma_s^{-1}]. \end{aligned} \quad (1)$$

The subscripts e, s and h represent the electrons, spinons and holons respectively, while the subscripts ab and c denote the ab-planes and c-axis respectively. γ and β however, are the experimental related constants of proportionality (range from $0 \rightarrow 1$), which are associated with the contribution of c-axis in ab-planes or vice versa [1, 2]. The term $\sigma_{s+h\rightarrow e}^{-1}$ is defined to be the resistivity caused by the $s + h \rightarrow e$ process occurring in the ab-planes that gives rise to the electron-electron scattering rate in ab-planes. If $s + h \rightarrow e$ is completely blocked in ab-planes then the electron's density in the ab-planes is zilch and consequently, $\sigma_{s+h\rightarrow e}^{-1} = 0$. Any increment in σ_e^{-1} also increases $\sigma_{s+h\rightarrow e}^{-1}$. Therefore, $\sigma_{s+h\rightarrow e}^{-1} \propto \sigma_e^{-1}$ (or $\sigma_{s+h\rightarrow e}^{-1} = \gamma\sigma_e^{-1}$).

In contrast, the term $\sigma_{e\rightleftharpoons s+h}^{-1}$ is defined to be the resistivity arises from the blockage in the $e \rightleftharpoons s + h$ processes. In other words, $\sigma_{e\rightleftharpoons s+h}^{-1}$ is due to the non-spontaneous $e \rightarrow s + h$ and $s + h \rightarrow e$ processes. These non-spontaneous processes imply that the spinons and holons are not energetically favorable in c-axis while the electrons are not energetically favorable in ab-planes. In addition, any increment in σ_{ab}^{-1} further increases the magnitude of the blockage in $e \rightleftharpoons s + h$ processes that eventually leads to a larger $\sigma_{e\rightleftharpoons s+h}^{-1}$. Consequently, $\sigma_{e\rightleftharpoons s+h}^{-1} \propto \sigma_{ab}^{-1}$ (or $\sigma_{e\rightleftharpoons s+h}^{-1} = \beta\sigma_{ab}^{-1}$). Simply put, the $e \rightleftharpoons s+h$ processes become increasingly difficult with increasing σ_{ab}^{-1}. This proportionality can also be interpreted as the additional scattering for the electrons to pass across ab-planes. If $e \rightleftharpoons s + h$ is spontaneous then $\sigma_{e\rightleftharpoons s+h}^{-1} = 0$.

2. Theoretical details

Here, Eq. (1) is shown to be microscopically relevant with the original Ioffe-Larkin's approach by using their effective long range action (S) in the presence of electromagnetic field (A), effective interactions between fermionic and bosonic fields and gauge field (a). The mentioned action that describes the Ioffe-Larkin formula is given by [3]

$$\begin{aligned} S\{A, a\} \;=\; \frac{T}{2}\int d\mathbf{k}\sum_{\omega}\big\{&[Q_sA(\omega,\mathbf{k}) + a(\omega,\mathbf{k})] \\ &\times\Pi_s(\omega,\mathbf{k})[Q_sA(\omega,\mathbf{k}) + a(\omega,\mathbf{k})] \\ &+[Q_hA(\omega,\mathbf{k}) + a(\omega,\mathbf{k})] \\ &\times\Pi_h(\omega,\mathbf{k})[Q_hA(\omega,\mathbf{k}) + a(\omega,\mathbf{k})]\big\}. \end{aligned} \quad (2)$$

The respective ω and $\mathbf{k}$ represent the frequency and the wave vector. Q_s and Q_h denote the arbitrary charges of a spinon and a holon respectively, while Q_e is the charge of an electron. Note that Eq. (2) ignores spinon pairing and arbitrary charges have been assigned accordingly [4]. Both spinons (fermions) and bosons (holons) interact with A and a. In this work, S in Eq. (2) is rewritten by writing an additional interaction generated by the electrons coexistence with spinons and holons, which is explicitly given by

$$\begin{aligned} \mathcal{S} = S\{A, a\} \;+\; \frac{T}{2}\int d\mathbf{k}\sum_{\omega}\big\{&Q_eA(\omega,\mathbf{k}) \\ &\times\Pi_{e\rightleftharpoons s+h}(\omega,\mathbf{k})Q_eA(\omega,\mathbf{k})\big\}. \end{aligned} \quad (3)$$

This additional term is zero to satisfy the principle of least action and also to imply that the electrons are not a separate entity in which, electrons flow is very much depends on spinons and holons flow and vice versa. This single-entity requirement will be discussed with appropriate limits shortly. The effective Lagrangian that corresponds to $\mathcal{S}$ is actually given by

$$
\begin{aligned}
\mathcal{L}[A,a] &= \sum_{ij} a_i(\Pi_s^{ij}+\Pi_h^{ij})a_j \\
&+2\sum_{ij} a_i(Q_s\Pi_s^{ij}+Q_h\Pi_h^{ij})A_j \\
&+\sum_{ij} A_i(Q_s^2\Pi_s^{ij}+Q_h^2\Pi_h^{ij})A_j \\
&+\sum_{ij} A_i\left[\frac{\big(\gamma+\beta\big)Q_e^2\Pi_s^{ij}\Pi_h^{ij}\Pi_e^{ij}}{\gamma\Pi_s^{ij}\Pi_h^{ij}+\beta\Pi_e^{ij}\Pi_s^{ij}+\beta\Pi_h^{ij}\Pi_e^{ij}}\right]A_j \\
&-\sum_{ij} A_i\left[\frac{\big(\gamma+\beta\big)Q_e^2\Pi_s^{ij}\Pi_h^{ij}\Pi_e^{ij}}{\gamma\Pi_s^{ij}\Pi_h^{ij}+\beta\Pi_e^{ij}\Pi_s^{ij}+\beta\Pi_h^{ij}\Pi_e^{ij}}\right]A_j. \quad (4)
\end{aligned}
$$

$\Pi_{s,h,e}$ denotes the response function for the spinons, holons and electrons, respectively. The single entity scenario allows electrons to pass across ab-planes with strong interaction with spinon-holon flow. On the contrary, if the electrons are an independent entity, not influenced by the spinons and holons flow, then the action, S_{ind} is simply given by

$$
\begin{aligned}
S_{ind} = S\{A,a\} \;+\; \frac{T}{2}\int d\mathbf{k}\sum_{\omega}\big\{Q_eA(\omega,\mathbf{k}) \\
\times\Pi_e(\omega,\mathbf{k})Q_eA(\omega,\mathbf{k})\big\}. \quad (5)
\end{aligned}
$$

Subsequently, the action, $\mathcal{S}$ can be averaged to arrive at

$$
\mathcal{S} = \frac{T}{2}\int d\mathbf{k}\sum_{ij}\big\{A_i(\omega,\mathbf{k})\Pi_{ij}A_j(\omega,\mathbf{k})\big\}. \quad (6)
$$

The averaging was carried out by utilizing the Gaussian integral [5], $\int \exp\big[-\big((1/2)(xWx)+Mx+N\big)\big]d^nx = \big((2\pi)^{n/2}/\sqrt{det\ W}\big)\exp\big[(1/2)MW^{-1}M-N\big]$. Therefore, the response function is given by

$$
\begin{aligned}
\Pi &= \frac{\big(\gamma+\beta\big)Q_e^2\Pi_s\Pi_h\Pi_e}{\gamma\Pi_s\Pi_h+\beta\Pi_e\Pi_s+\beta\Pi_h\Pi_e} \\
&-\frac{\big(\gamma+\beta\big)Q_e^2\Pi_s\Pi_h\Pi_e}{\gamma\Pi_s\Pi_h+\beta\Pi_e\Pi_s+\beta\Pi_h\Pi_e} \\
&+Q_s^2\Pi_s+Q_h^2\Pi_h-(Q_s\Pi_s+Q_h\Pi_h)^2(\Pi_s+\Pi_h)^{-1} \\
&= \frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h}\times \\
&\left[\frac{\big(\gamma+\beta\big)\Pi_e(\Pi_h+\Pi_s)-\big(\gamma+\beta\big)\Pi_e(\Pi_h+\Pi_s)}{\beta\Pi_e(\Pi_h+\Pi_s)+\gamma\Pi_h\Pi_s}\right] \\
&+\frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h} \\
&= \frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h}\times \\
&\left[\frac{\big(\gamma+\beta\big)\Pi_e(\Pi_h+\Pi_s)-\big(\gamma+\beta\big)\Pi_e(\Pi_h+\Pi_s)}{\beta\Pi_e(\Pi_h+\Pi_s)+\gamma\Pi_h\Pi_s}+1\right]. \quad (7)
\end{aligned}
$$

Firstly, if only spinons and holons exist in ab-planes where all $e\to s+h$, then Eq. (7) directly gives $\Pi = Q_e^2[\Pi_s^{-1}+\Pi_h^{-1}]^{-1}$. Simply put, the last two terms, in Eq. (4) which represent, $L_{e\rightleftharpoons s+h}$ equals 0, which in turn accentuates the pure CSS phenomenon. If only electrons exist in ab-planes, then $\Pi_h\Pi_s/(\Pi_h+\Pi_s)$ can be substituted with Π_e that eventually gives, $\Pi = Q_e^2\Pi_e$. Note that the above rearrangement of Eq. (7) using $\Pi_e = \Pi_h\Pi_s/(\Pi_h+\Pi_s)$ are *solely* to show that Eq. (7) is as it should be and does not violate the $e\rightleftharpoons s+h$ processes. In other words, the number of spinons and holons can only be increased with reduction in electron's number and $\Pi_e = \Pi_h\Pi_s/(\Pi_h+\Pi_s)$ has been employed *a priori*. Apart from the pure spinon-holon and pure electron phenomena, if one allows the coexistence of electrons with spinons and holons, then Eq. (7) can be reduced as

$$
\begin{aligned}
\Pi &= \frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h}\times \\
&\left[\frac{\big(\gamma+\beta\big)\Pi_e(\Pi_s+\Pi_h)-\big(\gamma+\beta\big)\Pi_e(\Pi_s+\Pi_h)}{\beta\Pi_e(\Pi_s+\Pi_h)+\gamma\Pi_s\Pi_h}+1\right] \\
&= \frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h}\times \\
&\left[\frac{\big(\gamma+\beta\big)\Pi_e(\Pi_s+\Pi_h)-\big(\gamma+\beta\big)\Pi_e(\Pi_s+\Pi_h)}{\beta\Pi_e(\Pi_s+\Pi_h)+\gamma\Pi_s\Pi_h}\right. \\
&\left.+\frac{\beta\Pi_e(\Pi_s+\Pi_h)+\gamma\big\{\Pi_s\Pi_h\big\}}{\beta\Pi_e(\Pi_s+\Pi_h)+\gamma\Pi_s\Pi_h}\right] \\
&= \frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h}\left[\frac{\big(\gamma+\beta\big)\Pi_e(\Pi_s+\Pi_h)}{\beta\Pi_e\Pi_s+\beta\Pi_e\Pi_h+\gamma\Pi_s\Pi_h}\right] \\
&= \frac{\big(\gamma+\beta\big)Q_e^2\Pi_s\Pi_h\Pi_e(\Pi_s+\Pi_h)}{(\Pi_s+\Pi_h)(\beta\Pi_e\Pi_s+\beta\Pi_e\Pi_h+\gamma\Pi_s\Pi_h)} \\
&= \frac{\big(\gamma+\beta\big)Q_e^2\Pi_s\Pi_h\Pi_e}{\beta\Pi_e\Pi_s+\beta\Pi_e\Pi_h+\gamma\Pi_s\Pi_h}. \\
&= \big(\gamma+\beta\big)Q_e^2[\beta\Pi_s^{-1}+\beta\Pi_h^{-1}+\gamma\Pi_e^{-1}]^{-1}. \quad (8)
\end{aligned}
$$

Notice that $\Pi_s\Pi_h$, indicated with $\{...\}$ in the fifth line has been substituted with $\Pi_e(\Pi_s+\Pi_h)$ that satisfies Ioffe-Larkin formula. This substitution means some of the electrons (Π_e) are converted to spinons (Π_s) and holons (Π_h) or vice versa, so as to allow the coexistence among electrons, spinons and holons (F-CSS liquid). After applying the linear-response theory, one can arrive at

$$
\sigma^{-1} \;=\; \beta\big[\sigma_s^{-1}+\sigma_h^{-1}\big]+\gamma\sigma_e^{-1}. \quad (9)
$$

$$
\sigma_{ab}^{-1} \;=\; \sigma_s^{-1}+\sigma_h^{-1}+\gamma\sigma_e^{-1}. \quad (10)
$$

$$\sigma_c^{-1} \ = \ \beta\big[\sigma_s^{-1}+\sigma_h^{-1}\big]+\sigma_e^{-1}. \tag{11}$$

Equations (10) and (11) are precisely in the form of Eq. (1), because $\beta = 1$ in ab-planes whereas $\gamma = 1$ in c-axis. Importantly, in ab-planes, $\gamma < 1$ and $\beta = 1$ whereas in c-axis, $\beta < 1$ and $\gamma = 1$. On the contrary, in the pure 2D CSS region with invalid $s + h \rightleftharpoons e$ processes, $\gamma = 0$ and $\beta = 1$ in ab-planes while $\gamma = 1$ and $\beta = 0$ in c-axis. Meaning, the spinons and holons that are confined in the ab-planes are literally independent of the electrons in c-axis, which automatically satisfies the original Ioffe-Larkin action given in Eq. (2). On the other hand, averaging the S_{ind} will lead one to the expressions

$$\Pi = \frac{Q_e^2\Pi_s\Pi_h}{\Pi_s+\Pi_h}+Q_e^2\Pi_e. \tag{12}$$

$$\sigma^{-1} = \frac{1}{\big[\sigma_s^{-1}+\sigma_h^{-1}\big]^{-1}+\sigma_e}. \tag{13}$$

Equation (13) implies that electrons flow is independent of spinons and holons.

3. Analysis

One can take the suitable limits, as given below in order to analyze the differences between Eqs. (9) and (13) respectively.

$$\lim_{\sigma_e^{-1}\to\infty}\sigma^{-1} = \infty, \ \lim_{\sigma_e^{-1}\to 0}\sigma^{-1} = \sigma_s^{-1}+\sigma_h^{-1}. \tag{14}$$

$$\lim_{\sigma_e^{-1}\to\infty}\sigma^{-1} = \sigma_s^{-1}+\sigma_h^{-1}, \ \lim_{\sigma_e^{-1}\to 0}\sigma^{-1} = 0. \tag{15}$$

Note that the stated Eqs. (14) and (15) are specifically for underdoped superconducting cuprates, however, the term σ_e^{-1} corresponds to the ionization energy based Fermi-Dirac statistics (iFDS). Equation (14) suggests that all components (electrons, spinons and holons) must superconduct so as to give a 3D superconductivity. Whereas, the limits in Eq. (15) point out that superconductivity can be achieved if any of the two phases (electron or spinon-holon) superconducts. The latter equation also implies that pure CSS is independently stable in 2D system, opposing the instability due to additional kinetic energy (KE) scenario calculated by Sarker [6]. Add to that, Varma *et al.* [7, 8] have also discussed that unlike in 1D, the conductivity of pure CSS phase in 2D is rather irreversible without additional KE. As for the overdoped cuprates, one can describe the transport properties namely, resistivity, Hall resistance and Lorenz ratio without employing the CSS mechanism. [9, 10, 11]. Basically, iFDS derived in the Refs. [9, 10, 11, 12, 13] has been employed for the latter work. Apart from that, iFDS is also found to be viable to determine the electronic properties of Ba-Sr-Ca-TiO_3 ferroelectrics [13], ferromagnets [14] and Carbon nanotubes [15].

In conclusion, two possible conductivity rules in the 2D superconducting systems have been discussed. Coexistence among spinons, holons and electrons requires their respective resistivities in series. In certain underdoped high-T_c cuprates, Eq. (9) is more appealing physically than Eq. (13).

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[1] A. Das Arulsamy, Physica B 352 (2004) 285.
[2] A. Das Arulsamy, P. C. Ong, M. T. Ong, Physica B 325 (2003) 164.
[3] L. B. Ioffe, A. I. Larkin, Phys. Rev. B 39 (1989) 8988.
[4] I. Ichinose, T. Matsui and M. Onoda, Phys. Rev. B 64 (2001) 104516.
[5] L. H. Ryder, Quantum Field Theory, Cambridge University Press, Cambridge, 1998.
[6] S. K. Sarker, Phys. Rev. B 68 (2003) 85105.
[7] C. M. Varma, Z. Nussinov, W. van Saarloos, Phys. Rep. 361 (2002) 267.
[8] B. Batlogg, C. M. Varma, Phys. World 13 (2000) 33.
[9] A. Das Arulsamy, Physica C 356 (2001) 62.
[10] A. Das Arulsamy, Phys. Lett. A 300 (2002) 691.
[11] A. Das Arulsamy, in: Paul S. Lewis (Ed.), Superconductivity Research at the Leading Edge, Nova Science Publishers, New York, 2004, pp. 45-57; A. Das Arulsamy, cond-mat/0408613.
[12] A. Das Arulsamy, cond-mat/0410443.
[13] A. Das Arulsamy, Phys. Lett. A 334 (2005) 413.
[14] A. Das Arulsamy, cond-mat/0406030.
[15] A. Das Arulsamy, cond-mat/0501008.